Solve for x:

1) $3^{2x+5} = 3^3$

$\log_3(3^{2x+5}) = \log_3 27$

$2x + 5 = 3$

$2x = -2$

$x = -1$

2) $5^{2x-5} = 125^{2x-11}$

$\log_5(5^{2x-5})$

$= \log_5(125^{2x-11})$

$2x - 5 = 3(2x - 11)$

$2x - 5 = 6x - 33$

$4x = 28$

$x = 7$

3) $\log x = 3$

$10^{\log x} = 10^3$

$x = 10^3$

$x = 1000$

4) $\log_5(x^2 + 1) = 1$

$5^{\log_5(x^2+1)} = 5^1$

$x^2 + 1 = 5$

$x^2 = 4$

$x = \pm 2$

5) $\log_x 32 = 5$

$x^{\log_x 32} = x^5$

$32 = x^5$

$x = \sqrt[5]{32}$

$x = 2$

6) $\log_4 4^{2x+3} = 9$

$2x + 3 = 9$

$2x = 6$

$x = 3$

7) $\log_{719}(5x - 8) = \log_{719}(2x + 7)$

$719^{\log_{719}(5x-8)} = 719^{\log_{719}(2x+7)}$

$5x - 8 = 2x + 7$

$3x = 15$

$x = 5$

8) $\log_3 180 - \log_3 6x = \log_3 4$

$\log_3 \frac{180}{6x} = \log_3 4$

$3^{\log_3 \frac{180}{6x}} = 3^{\log_3 4}$

$\frac{180}{6x} = 4$

$180 = 24x$

$x = \frac{15}{2}$

9) $\log_2 x = \frac{1}{4}\log_2 16 + \frac{1}{2}\log_2 49$

$\log_2 x = \log_2 \sqrt[4]{16} + \log_2 \sqrt{49}$

$\log_2 x = \log_2 2 + \log_2 7$

$\log_2 x = \log_2 14$

$2^{\log_2 x} = 2^{\log_2 14}$

$x = 14$

10) $\log_6(x + 2) + \log_6(x - 3) = 1$

$\log_6(x + 2)(x - 3) = 1$

$6^{\log_6(x^2-x-6)} = 6^1$

$x^2 - x - 6 = 6$

$x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

$x = 4, x = -3$

Reject $x = -3$, so $x = 4$ is the answer.

11) $7e^x - 3 = 0$

$7e^x = 3$

$e^x = \frac{3}{7}$

$\ln e^x = \ln \frac{3}{7}$

$x = \ln \frac{3}{7}$

12) $-4e^{5x} + 11 = 3$

$-4e^{5x} = -8$

$e^{5x} = 2$

$\ln e^{5x} = \ln 2$

$5x = \ln 2$

$x = \frac{\ln 2}{5}$

13) $2e^{2x-4} = 18$

$e^{2x-4} = 9$

$\ln e^{2x-4} = \ln 9$

$2x - 4 = \ln 9$

$2x = \ln(9) + 4$

$x = \frac{\ln(9) + 4}{2}$

14) $\ln \sqrt{x} = 4$

$e^{\ln \sqrt{x}} = e^4$

$\sqrt{x} = e^4$

$\sqrt{x}^2 = (e^4)^2$

$x = e^8$

15) $\ln(x - 3) = 1$

$e^{\ln(x-3)} = e^1$

$x - 3 = e$

$x = e + 3$

16) $\ln(x + 4) - 3 = 2$

$\ln(x + 4) = 5$

$e^{\ln(x+4)} = e^5$

$x + 4 = e^5$

$x = e^5 - 4$

Answer each question thoroughly.

Formulas: $y = ae^{rt},\ y = ae^{-rt},\ y = a(1+r)^t,\ y = a(1-r)^t,\ y = P\left(a + \frac{r}{n}\right)^{nt}$

17) Mr. Wytiaz discovers a banking error offering an annual rate of 87%, compounded continuously! He invests $20,000 of the math club budget. How much money will he have after 10 years?

$y = ae^{rt} \rightarrow y = 20000e^{.87(10)} = \$120058244.30$

18) In the future, America ceases funding the NASA program, choosing to focus on studying topsoil and how it can form dust clouds. Ms. Shaw reads about a spacecraft capable of interstellar flight for two people and suggests that Mr. Wytiaz spends $100,000,000 of the math club budget on the craft. He sees the benefits of being able to escape the planet for a while after assigning large sets of homework problems and agrees. They expect the spaceship to depreciate at a rate of 18% per year. What will the value of the spaceship be after 5 years?

$y = a(1-r)^t \rightarrow y = 100000000(1-.18)^5 = \$37073984.32$

39) Mr. Wytiaz and Ms. Shaw pilot a space exploration mission to gather data from a planet near a black hole. Unfortunately, Mr. Wytiaz separates one of the rockets too early and it crashes into a nuclear

power plant, setting off a chain reaction that wipes out all human life on Earth! Due to the effects of the black hole, they experience time at a different rate than they're used to and have no idea how much time has passed on Earth. When the math teachers return to Earth, they find the ruins of OHS. Mr. Wytiaz discovers some relatively undisturbed human remains clutching pieces of Laffy Taffy wrappers on the third floor of the building. He uses the instruments on his ship to analyze the deceased. He determines that only 1/100$^{\text{th}}$ of the amount of Carbon-14 (which has decay rate, $r = 0.00012$) that a living person would have remains. How long ago did the students perish?

$$y = ae^{-rt} \rightarrow \frac{1}{100} = 1e^{-.00012t}$$

$$\ln \frac{1}{100} = \ln e^{-.00012t}$$

$$\ln \frac{1}{100} = -.00012t$$

$$t = \frac{\ln \frac{1}{100}}{-.00012} = 38376.4\ yrs$$

40) Mr. Wytiaz discovers a secluded valley that saved a species of monkey from the near destruction of the planet. After learning to communicate with 100 monkeys and giving a few pre-tests, he believes that the primitive monkeys are not that different, cognitively, from the students he was teaching Secondary Math 3 Honors! Looking to resume his progress of building a world full of beings capable of complex problem-solving, he enrolls the monkeys in SM3H classes. At first, only 3 monkeys are able to solve word problems. Each week, the number of successful monkeys grows by 20%. How many weeks would it take for all 100 monkeys to be able to solve word problems?

$$y = a(1+r)^t \rightarrow 100 = 3(1+.2)^t$$

$$\frac{100}{3} = (1.2)^t$$

$$\ln \frac{100}{3} = \ln 1.2^t$$

$$\ln \frac{100}{3} = t \cdot \ln 1.2$$

$$t = \frac{\ln \frac{100}{3}}{\ln 1.2} = 19.2\ weeks$$

Previous Units Material that might show up on the test:

What is the remainder of each division problem?

41) $\dfrac{2x^4 + 3x^2 - x + 5}{x^2 + x + 1}$

$$\begin{array}{r} 2x^2 - 2x + 3 \\ x^2 + x + 1 \overline{\big| 2x^4 + 0x^3 + 3x^2 - x + 5} \\ \underline{-(2x^4 + 2x^3 + 2x^2)} \\ -2x^3 + x^2 - x \\ \underline{-(-2x^3 - 2x^2 - 2x)} \\ 3x^2 + x + 5 \\ \underline{-(3x^2 + 3x + 3)} \\ -2x + 2 \end{array}$$

Remainder: $-2x + 2$

42) $\dfrac{x^3 + 3x^2 + 2x + 1}{x + 7}$

$$\begin{array}{c|cccc} \boxed{-7} & 1 & 3 & 2 & 1 \\ & \downarrow & -7 & 28 & -210 \\ \hline & 1 & -4 & 30 & \boxed{-209} \end{array}$$

Remainder: -209

Simplify and state restrictions on x.

43) $\dfrac{x^2 - x - 6}{x^2 + 6x + 5} \div \dfrac{x^2 - 3x}{x^2 + 7x + 10}$

$$\frac{(x+2)(x-3)}{(x+5)(x+1)} \div \frac{x(x-3)}{(x+5)(x+2)} =$$

$$\frac{(x+2)(x-3)}{(x+5)(x+1)} \cdot \frac{(x+5)(x+2)}{x(x-3)} =$$

$$\frac{(x+2)(x+2)}{x(x+1)} =$$

$$\frac{x^2 + 4x + 4}{x^2 + x}, x \neq 0, -1, -5, 3$$

44) $\dfrac{2}{x+1} + \dfrac{x}{x-2}$

$LCD: (x+1)(x-2)$

$$\frac{2}{x+1} \cdot \frac{x-2}{x-2} + \frac{x}{x-2} \cdot \frac{x+1}{x+1} =$$

$$\frac{2x-4}{(x+1)(x-2)} + \frac{x^2 + x}{(x+1)(x-2)} =$$

$$\frac{x^2 + 3x - 4}{(x+1)(x-2)}, x \neq -1, 2$$